Integral Transforms (Prof. Dr. Jürgen Frikel)

Overview

This course provides a comprehensive study of fundamental integral transforms such as convolution, Fourier transform, and Fourier series, focusing on their exact mathematical treatment and practical applications. The course revisits essential concepts from functional analysis and integration theory to build a robust foundation for understanding these transforms.

Key Topics

1. **Review of Functional Analysis and Integration Theory:** Brief recap of core principles in functional analysis and central theorems of integration theory.

2. **Convolution on L^p-Spaces:**
   - Definition and mathematical properties of convolutions on L^p- spaces.
   - Functional approximation using convolutions, with a focus on Dirac sequences.
   - Importance of convolution in practical applications such as signal and image processing, deconvolution, and convolutional neural networks.

3. **Fourier Transform:**
   - Fourier transform on L^1
   - Extension of the Fourier transform to L^2
   - Applications in signal and image processing, including filtering and deconvolution.
   - Application to partial differential equations, with a detailed discussion on the heat equation.

4. **Fourier Series:**
   - Real and complex forms of Fourier series.
   - Pointwise and uniform convergence of Fourier series.
   - Fourier series in Hilbert spaces.
   - Practical applications of Fourier series.

5. **Discrete Fourier Transform:** Fundamental concepts and applications.

6. **Optional Topics:** Time-frequency analysis, such as Short-Time Fourier Transform (windowed Fourier transforms) and/or wavelets, and their applications.

Learning goals: By the end of this course, students will:
- Understand the mathematical foundations of convolution and Fourier transforms.
- Be able to apply convolution and Fourier transforms in various practical scenarios, particularly in signal and image processing.
- Gain insight into the use of Fourier transforms in solving partial differential equations.
- Develop proficiency in working with Fourier series and discrete Fourier transforms.
- Explore advanced topics like time-frequency analysis and wavelets, if covered.